9.4 Graph and Write Equations of Ellipses

Goal • Graph and write equations of ellipses.

Your Notes

VOCABULARY

Ellipse The set of all points P such that the sum of the distances between P and two fixed points, called the foci, is a constant

Foci Two fixed points in an ellipse

Vertices The points at which the line through the foci intersect the ellipse

Major axis The line segment that joins the vertices

Center The midpoint of the major axis

Co-vertices The points of intersection of an ellipse and the line perpendicular to the major axis at the center

Minor axis The line segment that joins the co-vertices

STANDARD EQUATION OF AN ELLIPSE WITH CENTER AT THE ORIGIN

Equation	Major Axis	Vertices	Co-Vertices
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	Horizontal	$(\pm a, 0)$	$(0, \pm b)$
$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$	Vertical	$(0, \pm a)$	$(\pm b, 0)$

The major and minor axes are of lengths $2a$ and $2b$, respectively, where $a > b > 0$. The foci of the ellipse lie on the major axis at a distance of c units from the center, where $c^2 = a^2 - b^2$.

Your Notes

Example 1 ***Graph an equation of an ellipse***

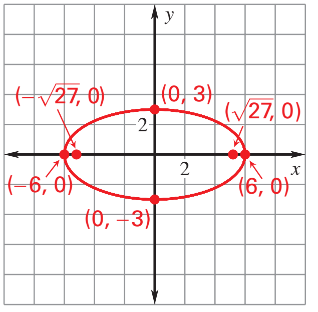
Graph the equation $9x^2 + 36y^2 = 324$. Identify the vertices, co-vertices, and foci of the ellipse.

1. Rewrite the equation in standard form.

$$9x^2 + 36y^2 = 324 \qquad \textbf{Write original equation.}$$

$$\frac{9x^2}{342} + \frac{36y^2}{324} = \frac{324}{324} \qquad \textbf{Divide each side by } \underline{324}.$$

$$\frac{x^2}{36} + \frac{y^2}{9} = 1 \qquad \textbf{Simplify.}$$



2. Identify the vertices, co-vertices, and foci. Note that $a^2 = \underline{36}$ and $b^2 = \underline{9}$, so $a = \underline{6}$ and $b = \underline{3}$. The denominator of the x^2-term is <u>greater than</u> that of the y^2-term, so the major axis is <u>horizontal</u>. The vertices of the ellipse are at $(\pm a, 0) = (\pm \underline{6}, 0)$. The co-vertices are at $(0, \pm b) = (0, \pm \underline{3})$. Find the foci.

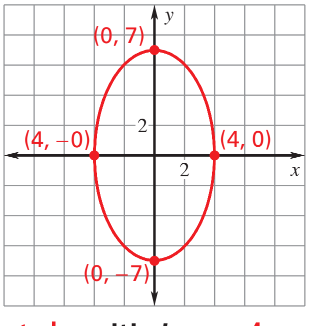
$c^2 = a^2 - b^2 = \underline{6^2 - 3^2} = \underline{27}$, so $c = \sqrt{\underline{27}}$.

The foci are at $(\pm\sqrt{\underline{27}}, 0)$, or about $(\pm \underline{5.2}, 0)$.

3. Draw the ellipse that passes through each vertex and co-vertex.

Example 2 ***Write an equation given a vertex and a co-vertex***

Write an equation of the ellipse that has a vertex at (0, 7), a co-vertex at (−4, 0), and center at (0, 0).



Sketch the ellipse as a check for your final equation. By symmetry, the ellipse must also have a vertex at $(0, \underline{-7})$ and a co-vertex at $(\underline{4}, 0)$.

Because the vertex is on the <u>y-axis</u> and the co-vertex is on the <u>x-axis</u>, the major axis is <u>vertical</u> with $a = \underline{7}$, and the minor axis is <u>horizontal</u> with $b = \underline{4}$.

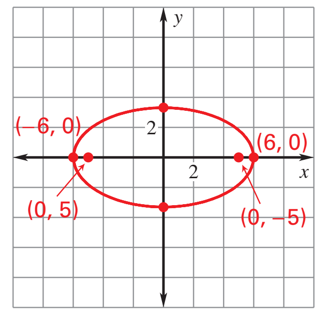
An equation is $\underline{\dfrac{x^2}{4^2} + \dfrac{y^2}{7^2}} = 1$, or $\underline{\dfrac{x^2}{16} + \dfrac{y^2}{49}} = 1$.

Your Notes

Example 3 ***Write an equation given a vertex and a focus***

Write an equation of the ellipse that has a vertex at $(-6, 0)$ and a focus at $(5, 0)$.

Solution



Make a sketch of the ellipse. Because the vertex and focus lie on the x-axis, the major axis is horizontal, with $a = 6$ and $c = 5$. To find b, use the equation $c^2 = a^2 - b^2$.

$$5^2 = 6^2 - b^2$$

$$b^2 = 6^2 - 5^2 = 11$$

$$b = \sqrt{11}$$

An equation is $\dfrac{x^2}{6^2} + \dfrac{y^2}{(\sqrt{11})^2} = 1$, or $\dfrac{x^2}{36} + \dfrac{y^2}{11} = 1$.

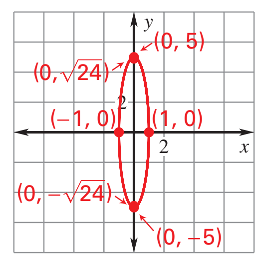
Checkpoint **Graph the equation. Identify the vertices, co-vertices, and foci of the ellipse.**

1. $x^2 + \dfrac{y^2}{25} = 1$

vertices: $(0, \pm 5)$

co-vertices: $(\pm 1, 0)$

foci: $(0, \pm 4.90)$



Checkpoint **Write an equation of the ellipse with the given characteristics and center at $(0, 0)$.**

2. Vertex: $(-9, 0)$
Co-vertex: $(0, 4)$

$\dfrac{x^2}{81} + \dfrac{y^2}{16} = 1$

3. Vertex: $(0, 7)$
Focus: $(0, -3)$

$\dfrac{x^2}{40} + \dfrac{y^2}{49} = 1$

Homework